Small-Hole Formalism for the FDTD Simulation of Small-Hole Coupling

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Abstract—**Based on Bethe's small hole coupling theory, a small-hole formalism (SHF) for the finite-difference time-domain simulation of small hole coupling is presented and validated. Much of the computing time and computer resources are saved by the SHF presented here.**

I. Introduction

THE finite-difference time-domain (FDTD) method is widely used for modeling the electromagnetic response of systems [1], [2]. In the modeling of small hole coupling, one must either reduce the spatial cell size down to that required to resolve the hole, or adopt an alternative method to characterize the hole. In this letter, a small-hole formalism (SHF) for the FDTD simulation of small hole coupling is presented and validated to be useful for the FDTD simulation of small hole coupling.

II. Small-Hole Formalism

Without losing generality, the example shown in Fig. 1 is studied. Also shown in Fig. 1 is the typical Yee's FDTD mesh just adjacent to the small hole. Based on Bethe's small hole coupling theory [3] and the field equivalence theorem [4], in the vicinity of the hole, the total fields in region 1 and region 2 can be calculated by

$$\mathbf{E}_1 = \mathbf{E}_0 + \mathbf{E}_1^d(-\mathbf{p}/2, -\mathbf{m}/2), \quad \mathbf{H}_1 = \mathbf{H}_0 + \mathbf{H}_1^d(-\mathbf{p}/2, -\mathbf{m}/2) \tag{1}$$

$$\mathbf{E}_2 = \mathbf{E}_2^d(\mathbf{p}/2, \mathbf{m}/2), \quad \mathbf{H}_2 = \mathbf{H}_2^d(\mathbf{p}/2, \mathbf{m}/2) \tag{2}$$

where ($\mathbf{E}_0$, $\mathbf{H}_0$) is the original field before the hole is cut in the wall, ($\mathbf{E}_{1,2}^d$, $\mathbf{H}_{1,2}^d$) are the fields of the equivalent electric and magnetic dipoles ($\mp\mathbf{m}/2$, $\mp\mathbf{p}/2$) located at the center of the hole. $\mathbf{m}$ and $\mathbf{p}$ are related to ($\mathbf{E}_0$, $\mathbf{H}_0$) by

$$\mathbf{p} = \varepsilon_0 P E_{0n}\mathbf{n}, \quad \mathbf{m} = -M_u H_{0u}\mathbf{u} - M_v H_{0v}\mathbf{v} \tag{3}$$

where $\mathbf{n}$ is the unit normal vector of the hole plane, $\mathbf{u}$ and $\mathbf{v}$ are unit vectors along the two orthogonal principal axes of the hole at the center of the hole. The electric and magnetic polarizabilities P, M_u, and M_v are functions only of the shape and size of the hole and can be determined by analytic methods [3] or by electrolytic-analog measurements [5].

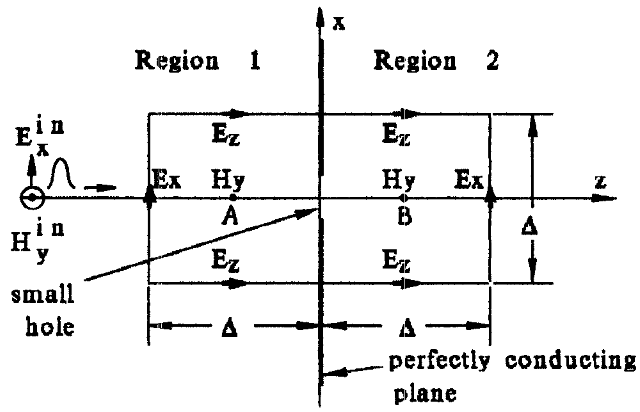

Fig. 1. Small-hole coupling and the typical Yee's FDTD mesh near the hole.

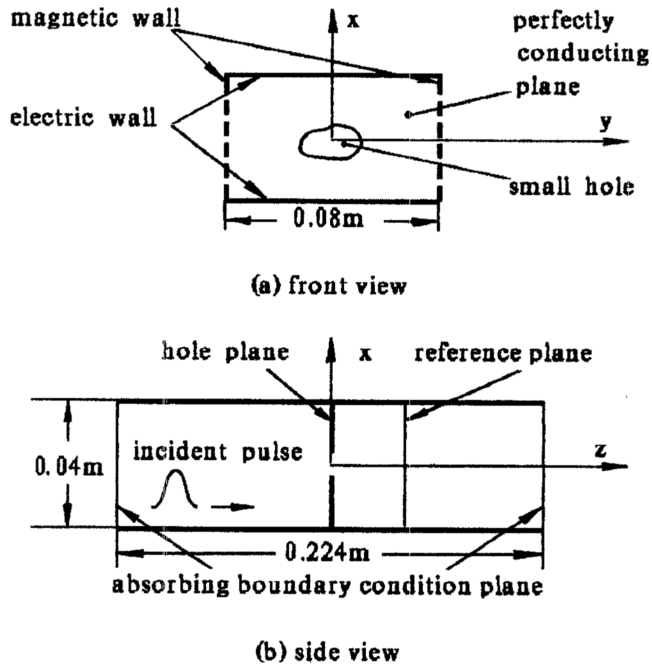

Fig. 2. Example of small-hole coupling.

Since $E_{0n} = 0$, only a magnetic dipole, $\mathbf{m}/2 = -\mathbf{a}_y (M/2)H_{0y,\text{hole}}$, exists. For the original field, there is no hole, and the magnetic field component at the hole can be approximated by the magnetic field component at node **A**,

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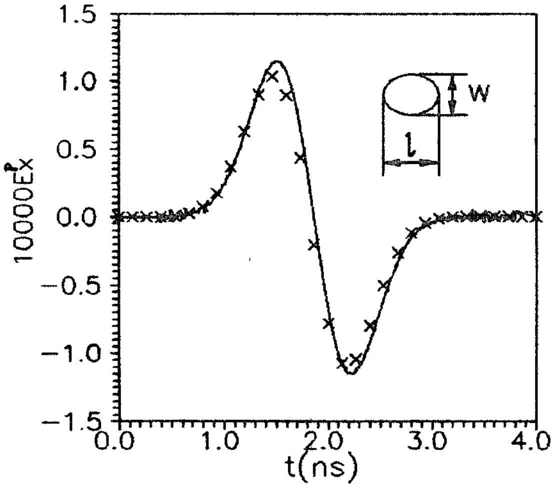

Fig. 3. Waveforms of the penetrating electric field E_x^p for an elliptical hole ($w/l = 0.75$, $w = 0.006$ m, $M/l^3 = 0.135$ [5]). × × ××: SHF with a spatial step of $h = 0.016$ m; —: ordinary FDTD with a much finer spatial step of $h/16$.

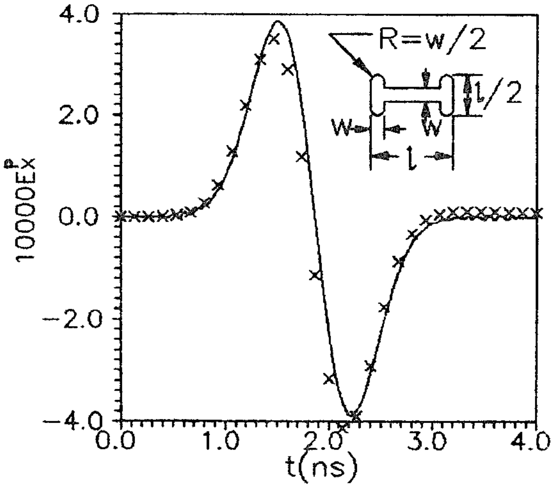

Fig. 5. Waveforms of the penetrating electric field E_x^p for a H-shaped hole ($w/l = 1/6$, $w = 0.002$, $M/l^3 = 0.144$ [5]). × × ××: SHF with a spatial step of $h = 0.016$ m; —: ordinary FDTD with a much finer spatial step of $h/16$.

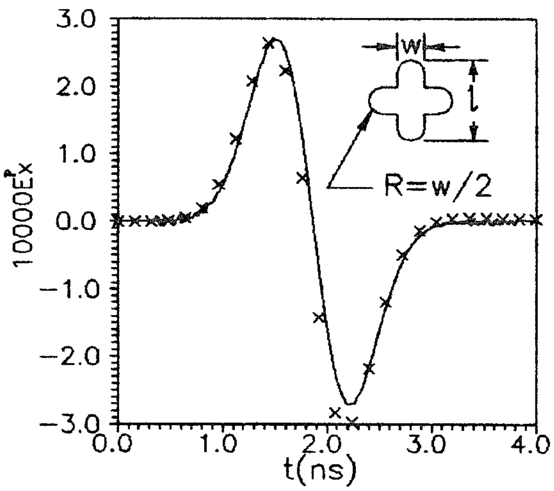

Fig. 4. Waveforms of the penetrating electric field E_x^p for a cross-shaped hole ($w/l = 1/3$, $w = 0.004$ m, $M/l^3 = 0.107$ [5]). × × ××: SHF with a spatial step of $h = 0.016$ m; —: ordinary FDTD with a much finer spatial step of $h/16$.

i.e., $\mathbf{H}_{0y,hole} \approx \mathbf{H}_{0y,\mathbf{A}}$. At node A and node B, which are the nearest nodes to the hole, according to (1), (2), and the well-known magnetic dipole field formula, we get the following SHF which is compatible with the ordinary FDTD algorithm,

$$\begin{aligned} H_{y,A} &= H_{0y,A} - \frac{M}{8\pi(\Delta/2)^3} H_{0y,\text{hole}} \\ &\approx H_{0y,A} - \frac{M}{8\pi(\Delta/2)^3} H_{0y,A} \end{aligned} \qquad (4)$$

$$H_{y,B} = \frac{M}{8\pi(\Delta/2)^3} H_{0y,\text{hole}} \approx \frac{M}{8\pi(\Delta/2)^3} H_{0y,A} \qquad (5)$$

where $H_{0y,A}$ can be obtained by ordinary FDTD equations. At the other nodes, the ordinary FDTD equations are used directly for the total fields ($\mathbf{E}_1$, $\mathbf{H}_1$) in region 1 and ($\mathbf{E}_2$, $\mathbf{H}_2$) in region 2. The perturbed dipole fields at node A and node B, included in (4) and (5) respectively, will propagate to the other nodes through a standard FDTD simulation procedure.

III. Numerical Results

To validate the SHF outlined above, the example shown in Fig. 2 is simulated by incorporating the SHF into the ordinary FDTD algorithm. A TEM mode Gaussian pulse, $E_x^{\text{in}} = \exp\left[-(n\Delta t - t_0)^2/T^2\right]$, is applied as the incident pulse, where $t_0 = 3T$, $T = 0.5$ ns, and the effective frequency spectrum of the pulse ranges from DC to 1 GHz. The penetrating electric field component E_x^p is monitored at the middle point of the reference plane which is 0.064 m away from the hole plane.

Figs. 3–5 show the waveforms of E_x^p calculated by the SHF with a spatial step of $h = \Delta x = \Delta y = \Delta z = 0.016$ m and a time step $\Delta t = h/2c$ where c is the speed of light in free space. In Figs. 3–5, the small holes are, respectively, elliptical, cross-shaped, and H-shaped apertures. The results by an ordinary FDTD algorithm (without SHF) with a much finer resolution of $h/16$ are also shown in Figs. 3–5, and display a good agreement with those by the SHF, while much of the computing time and computer resources are saved by the SHF.

IV. Conclusion

The SHF presented here has been validated to be useful for the FDTD simulation of small hole coupling. For other small holes with shapes different from the examples given above, the only difference is that they have some different polarizabilities P, M_u, and M_v, which can be determined by analytic methods [3] or by electrolytic-analog measurements [5], while the FDTD simulation procedure is the same.

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